terizing sonic booms. Signal durations of up to 75 msec were recorded at distances less than 800 ft from the point of balloon detonation. Peak overpressures in the range 3–15 psf were obtained.

The detonations of gas-filled, slender, shaped balloons of the particular geometry tested in this project yield an N-wave at moderate ranges from the source. These balloon detonations could be used directly as pressure signal inputs to a number of sonic boom effect studies. Response studies of structures such as existing buildings and full-scale building components, for example, an exterior wall specimen containing windows and window casements, can be readily and inexpensively tested compared to overflight testing. Since the balloons can be fabricated to any desired size scale, model studies can also be performed. Certain physiological testing might also be considered as this technique generates a signal which does not have much high frequency noise superposed on the signal, a characteristic similar to sonic booms not exhibited by solid explosive simulation.

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Downwash-Velocity Potential Method for Lifting Surfaces

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Introduction

THIS Note presents some preliminary calculations of subsonic forces on rectangular wings, using the downwash-velocity potential relationship. Generally, the downwash-pressure method has been most widely used, as for example by Watkins[1] and Albano and Rodden.[2] However, Jones,[3] Stark,[4] and Houbolt[5] have used the velocity potential in their methods. Only the steady-state case is considered here.

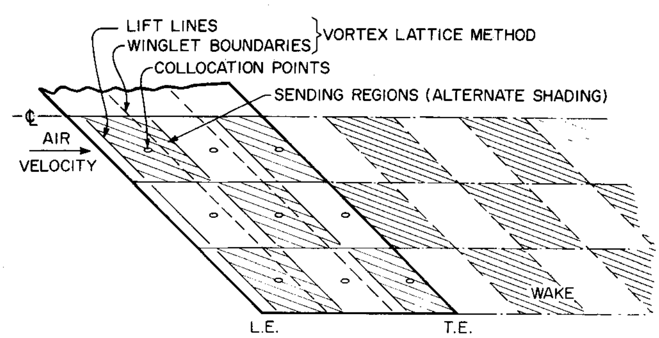

Fig. 1 Typical wing planform showing equivalent vortex-lattice array.

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Table 1 Comparison with vortex lattice method

	Vortex lattice	These results
Aspect ratio 2	$C_{L\alpha} = 3.10902$	3.10899
	$C_{M\alpha} = -1.26869$	-1.26828
Aspect ratio 1	$C_{L\alpha} = 1.75507$	1.75508
	$C_{M\alpha} = -0.53027$	-0.53027

Derivation

Representing an airfoil by a flat surface parallel to the x,y plane, with a given normal velocity component w, the unknown velocity potential jump $\Delta\phi$ is given by the integral equation:

$$w(x,y) = \iint \Delta\phi(\xi,\eta)\partial^2/\partial z^2\{1/4\pi R\}d\xi d\eta \tag{1}$$

where integration is over the wing and wake, with

$$R^2 = (x-\xi)^2 + \alpha^2(y-\eta)^2 + \alpha^2(z-\zeta)^2$$

and $\alpha^2 = 1 - M^2$ (M = Mach number).

Selecting a set of collocation points and denoting "receiving" points by k, "sending" points by l, the assumption of uniform velocity potential over a sending area region near l leads to the following expression for the downwash at k:

$$w_k = \Sigma(l'')\Delta\phi_l''k_\phi(k,l'') + \Sigma(l')\Sigma(l^*) \times \exp\{i\omega(x_l' - x_l^*)/V\}\Delta\phi_l'k_\phi(k,l^*) = \Sigma(l)A_{k,l}\Delta\phi_l \tag{2}$$

where l' represents a point on the trailing edge, and l'' one of the remaining collocation points, while l^* is a point in the wake that is not one of the collocation points. $A_{k,l}$ is the aerodynamic matrix and k_ϕ is the airforce influence coefficient, given by

$$k_\phi(k,l) = \iint_{\text{region } l}[\alpha^2/4\pi R^3(k)]d\xi d\eta \tag{3}$$

where $R(k)$ is R evaluated at the point k, with coordinates (x_k,y_k). Summation over l^* in Eq. (2) is limited to a finite distance into the wake. An alternate form of Eq. (2) replaces the wake integration by lifting lines along the trailing edge.

For rectangular sending regions having dimensions L_X by L_Y, with the point l (coordinates ξ_l,η_l) at their centers, Eq. (3) can be integrated exactly to give

$$k_\phi(k,l) = F(\xi_l + \tfrac{1}{2}L_X,\eta_l + \tfrac{1}{2}L_Y) - F(\xi_l + \tfrac{1}{2}L_X,\eta_l - \tfrac{1}{2}L_Y) - F(\xi_l - \tfrac{1}{2}L_X,\eta_l + \tfrac{1}{2}L_Y) + F(\xi_l - \tfrac{1}{2}L_X,\eta_l - \tfrac{1}{2}L_Y) \tag{4}$$

where $F(X,Y) = \{(x_k - X)^2 + \alpha^2(y_k - Y)^2\}^{1/2}/4\pi(x_k - X)(y_k - Y)$

Given that the local normal velocity equals the local angle of attack multiplied by the velocity, the velocity potentials in Eq. (2) can be solved by simultaneous equations. From these section lifts and moments can be calculated.

Preliminary Calculations

These were made on rectangular planforms at uniform angle of attack. In order to get direct verification of the method, the rectangular sending regions were oriented in such a way

Table 2 Effect of wake length[a]

Wake, chord lengths	$C_{L\alpha}$	$C_{M\alpha}$
1	2.6263	-1.0992
5	2.8390	-1.1886
10	2.8507	-1.1935
50	2.8547	-1.1952
100	2.8548	-1.1952
500	2.8549	-1.1953

[a] Note convergence to within 1% in five chord lengths.

Table 3 Effect of Array Size

Array size	$C_{L\alpha}$	$C_{M\alpha}$	$c.p. = -C_{M\alpha}/C_{L\alpha}$
2×2	3.0947	−1.3683	0.4422
3×3	2.9374	−1.2507	0.4258
4×4	2.8548	−1.1952	0.4187
6×6	2.7695	−1.1427	0.4126
8×8	2.7259	−1.1176	0.4100

that the vortex lattice method of Hedman[6] was simulated, as illustrated in Fig. 1. The number of regions in the wake was limited to so many chord lengths. A summary of results follows.

Check against Hedman's results

Two rectangular planforms were analyzed at a Mach Number of 0.7, with a wake of 100 chord lengths. The first had an aspect ratio of 2, with a 3×3 array of sending regions (boxes) on each half wing. The second had an aspect ratio of unity, with a 6 (chordwise) $\times$ 3 (spanwise) array on each half wing.

Corresponding results according to Hedman's[6] vortex lattice method were obtained using a computer program based on the method of Albano and Rodden,[2] that computes the steady state component of the aerodynamic matrix by Hedman's method. Results are shown in Table 1.

The agreement shown in Table 1 is within the convergence limits reported in Table 2 for a wake of 100 chord lengths.

Effect of wake length

The effect of finite wake length was investigated for a wing with an aspect ratio of 2, and a 4×4 array of sending points on the half wing, at a Mach Number of 0.5. Results are shown in Table 2.

Effect of array size

The effect of array size was investigated on a wing with an aspect ratio of 2, at a Mach number of 0.5, with a wake of 100 chord lengths. Results are shown in Table 3.

It is interesting to note that $C_{L\alpha}$ and *c.p.* have already converged to within 10% with a 2×2 array, and to within 5% for a 4×4 array. Previously, a midpoint calculation had been tried for the airforce influence coefficients that resulted in

$$k_\phi(k,l) = -\alpha^2 L_X L_Y / 4\pi R^3 \tag{5}$$

except for the (O,O) case, where the expression given in Eq. (4) was used. The results of the two sets of calculations are plotted on a logarithmic scale in Fig. 2. They indicate convergence when Eq. (4) is used, but non-convergence when the approximate form of Eq. (5) is used.

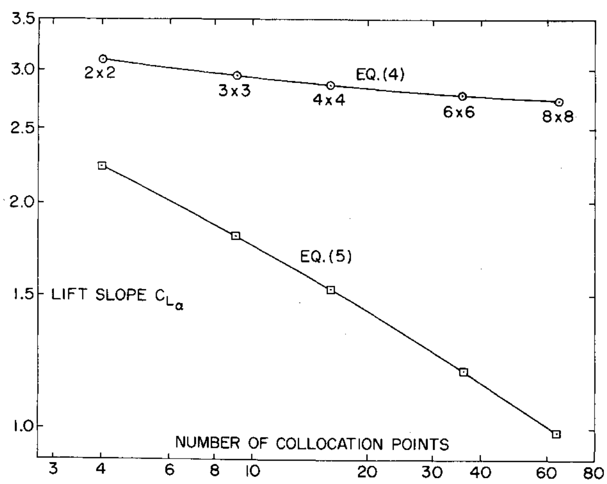

Fig. 2 **Lift slope $C_{L\alpha}$ vs number of collocation points.**

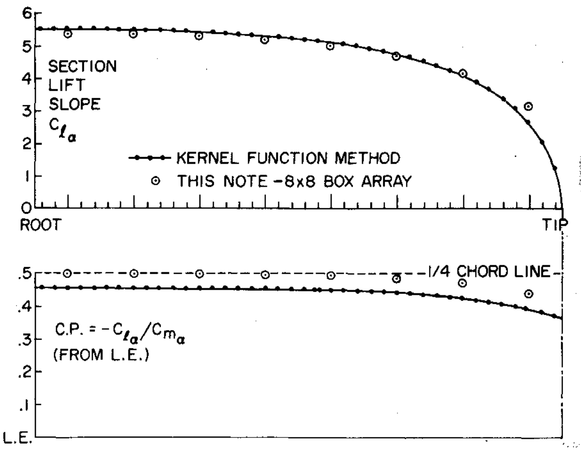

Fig. 3 Comparison with kernel function method; $M = 0$, and aspect ratio = 8.

Comparison with the kernel function method

A comparison was made with Watkin's[1] kernel function method at a Mach number of zero on a wing with an aspect ratio of 8. An 8×8 array was used with a wake of 100 chord lengths. The kernel function results were obtained for 16 collocation points.

The result is shown in Fig. 3. Agreement is generally fair, the worst point being the center of pressure location at the root of the wing. The near $\frac{1}{4}$ chord results of the new method appear to be more correct than those given by the kernel function method.

Computer time

The CPU time used on the CDC 6400 computer for the 8×8 array with a wake of 100 chord lengths was 99 sec. This involved 64 receiving points and approximately 1600 sending points, requiring about 102,400 applications of Eq. (4). Finally, a 64×64 matrix was inverted.

Conclusions

It has been demonstrated in the steady-state case that the method is capable of achieving 5% accuracy with moderate array sizes, involving 16 points or less, and with wake of less than 5 chord lengths. Further, it has been shown that the accuracy improves as these numbers increase. Future work will investigate the applicability of an extended method to the oscillating case with arbitrary planforms.

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